8. A curve C with equation $y = f(x)$ passes through the point $R(4, 13)$.

Given that

$$f'(x) = 2(x - 3)(3x + 2)$$

(a) use integration to find $f(x)$, giving your answer in simplest form.

(5)

(b) Given that $f(x)$ can be written in the form

$$(x - 3)^2(px + q)$$

find the value of the constant p and the value of the constant q.

(2)

(c) Sketch the graph of $y = f(2x)$, showing the coordinates of any points where the curve touches or crosses the coordinate axes.

(4)

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 11 marks)

9.

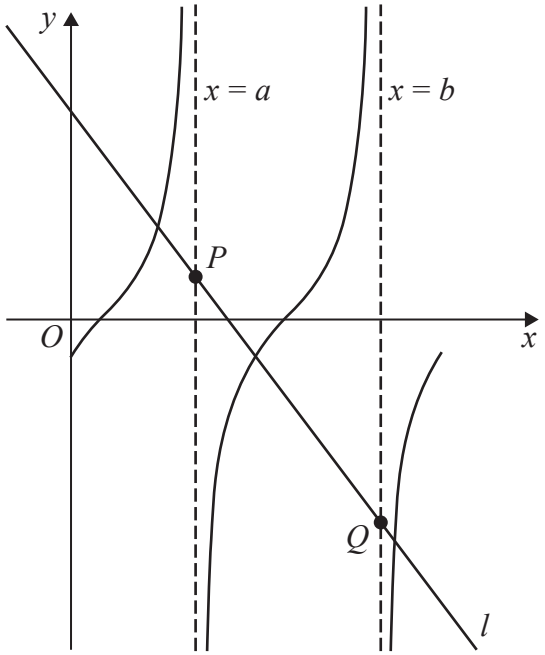


Figure 3

Figure 3 shows a sketch of

- the curve with equation $y = \tan\left(x - \frac{\pi}{6}\right)$ for $0 \leqslant x \leqslant 2\pi$
- part of the straight line l with equation $y = \pi - x$

(a) State the number of solutions of the equation

(i) $\tan\left(x - \frac{\pi}{6}\right) = \pi - x$ in the interval $0 \leqslant x \leqslant 2\pi$

(ii) $\tan\left(x - \frac{\pi}{6}\right) = \pi - x$ in the interval $0 \leqslant x \leqslant 100\pi$

(iii) $\tan\left(x - \frac{\pi}{6}\right) = \pi + x$ in the interval $0 \leqslant x \leqslant 2\pi$

(3)

The line with equation $x = a$, shown in Figure 3, is the asymptote to the curve with the smallest positive x coordinate.

(b) State the value of a

(1)

The line with equation $x = b$, also shown in Figure 3, is the asymptote to the curve with the second smallest positive x coordinate.

The line l meets $x = a$ at point P and meets $x = b$ at point Q as shown in Figure 3.

(c) Find the midpoint of the line segment PQ.

(4)

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