Alternatives for part (ii)

(ii) ALT 1	$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow\left(p+q\sqrt{3}\right)\sqrt{3}-3=p+q\sqrt{3}+\sqrt{3}\Rightarrow p\sqrt{3}+3q-3=p+\sqrt{3}+q\sqrt{3}$ $\Rightarrow p=q+1,\ 3q-3=p$	M1
	$\Rightarrow p=q+1,\ 3q-3=p\Rightarrow p=...,q=...$	M1
	$x=3+2\sqrt{3}$	A1

Notes:

M1: Substitutes $x=p+q\sqrt{3}$, collects terms, compares coefficients and forms 2 equations in p and q
It is for obtaining 2 equations of the form $\alpha p+\beta q=\gamma,\ \alpha,\beta,\gamma\neq 0$

M1: Solves simultaneously with evidence of algebra i.e. answers not just written down from a calculator.
Not formally dependent but requires the solution of 2 equations of the form $\alpha p+\beta q=\gamma,\ \alpha,\beta,\gamma\neq 0$

A1: For $3+2\sqrt{3}$. Allow $2\sqrt{3}+3$. It is for this expression and not for just p = 3, q = 2 unless $3+2\sqrt{3}$ is seen.

(ii) ALT 2	$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow 3x^2-6\sqrt{3}x+9=x^2+2\sqrt{3}x+3$ $\Rightarrow 2x^2-8\sqrt{3}x+6=0$	M1
	$x^2-4\sqrt{3}x+3=0\Rightarrow x=\dfrac{4\sqrt{3}\pm\sqrt{\left(4\sqrt{3}\right)^2-4\times1\times3}}{2}$	M1
	$\Rightarrow x=\dfrac{4\sqrt{3}\pm\sqrt{36}}{2}\Rightarrow x=2\sqrt{3}\pm3$ $x=2\sqrt{3}+3$	A1

Notes:

M1: Squares both sides. Must obtain a 3 term quadratic expression on both sides and collect terms to one side.

M1: Solves a 3TQ of the form $px^2+q\sqrt{3}x+r=0$ by a correct non-calculator method.

A1: For $3+2\sqrt{3}$. Allow $2\sqrt{3}+3$. It is for this expression and not for just p = 3, q = 2 unless $3+2\sqrt{3}$ is seen.
If any other answers are offered and clearly not rejected score A0.

(ii) ALT 3	$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow\left(x\sqrt{3}-3\right)\left(x-\sqrt{3}\right)=\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$ $\Rightarrow\sqrt{3}x^2-6x+3\sqrt{3}=x^2-3\Rightarrow\left(\sqrt{3}-1\right)x^2-6x+3\sqrt{3}+3=0$	M1
	$\left(\sqrt{3}-1\right)x^2-6x+3\sqrt{3}+3=0\Rightarrow x=\dfrac{6\pm\sqrt{36-4\left(\sqrt{3}-1\right)\left(3\sqrt{3}+3\right)}}{2\left(\sqrt{3}-1\right)}$ $=\dfrac{6\pm\sqrt{36-24}}{2\left(\sqrt{3}-1\right)}=\dfrac{6\pm\sqrt{12}}{2\left(\sqrt{3}-1\right)}=\dfrac{3\pm\sqrt{3}}{\sqrt{3}-1}=\dfrac{3\pm\sqrt{3}}{\sqrt{3}-1}\times\dfrac{\sqrt{3}+1}{\sqrt{3}+1}$	M1
	$=\dfrac{3\sqrt{3}+3\pm3\pm\sqrt{3}}{2}=2\sqrt{3}+3,\ \sqrt{3}$ $x=2\sqrt{3}+3$	A1

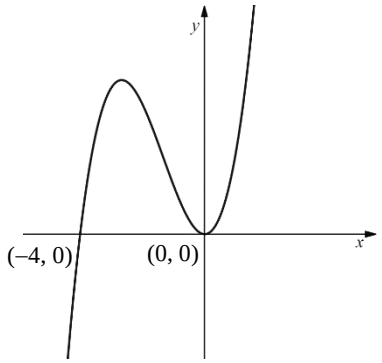
Notes:

M1: Multiples both sides by e.g. $\left(x-\sqrt{3}\right)$ and collects terms to one side.
Must obtain a 3 term quadratic expression on lhs and a 2 or 3 term quadratic expression on rhs.

M1: Solves a 3TQ of the form $\left(A+B\sqrt{3}\right)x^2+Cx+D+E\sqrt{3}=0,\ A,B,C,D,E\neq0$ by a correct non-calculator method **and** then attempts to rationalise the denominator as in the main scheme.

A1: For $3+2\sqrt{3}$. Allow $2\sqrt{3}+3$. It is for this expression and not for just p = 3, q = 2 unless $3+2\sqrt{3}$ is seen.
If any other answers are offered and clearly not rejected score A0.

		Total 6

Question	Scheme	Marks
3(a)		B1B1B1
		(3)

Notes:

If there is any ambiguity regarding coordinates, the sketch takes precedence.

B1: Same shape translated parallel to the *x*-axis with no evidence **from the *x*-intercepts** (if labelled) that the transformation is anything other than a translation. So the *x* intercepts (if labelled) must be 4 units apart. Ignore any coordinates on or near the maximum.

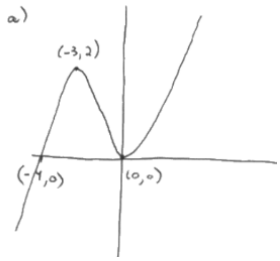
B1: Minimum at the origin. The origin does not have to be labelled.

B1: Passes through (−4, 0) or at least "reaches" this point from above or below.

Allow just −4 marked in the correct place and condone (0, −4) as long as it is marked in the correct place.

Allow to score away from the sketch as long as their curve passes through (or reaches) this point but must be identified correctly as (−4, 0) and correspond to the sketch.

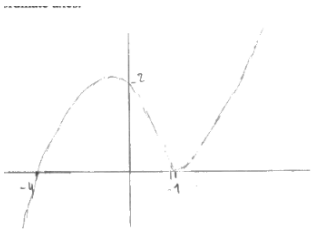
See examples below:



B1: Translation (the maximum at (−3, 2) can be ignored)

B1: Minimum at the origin

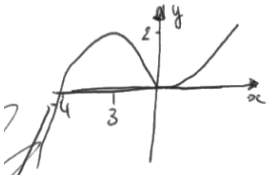
B1: Passes through (−4, 0)



B0: The *x* intercepts are contradictory for a translation (not 4 units apart)

B0: No minimum at the origin

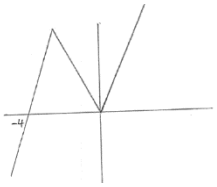
B1: Passes through (−4, 0)



B1: Translation

B1: Minimum at the origin (the intention is clear)

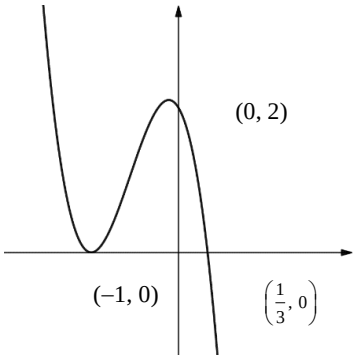
B1: Passes through (−4, 0)



B0: Incorrect shape

B1: Minimum at the origin

B1: Passes through (−4, 0)

(b)		B1B1B1
		(3)
		Total 6

Notes:

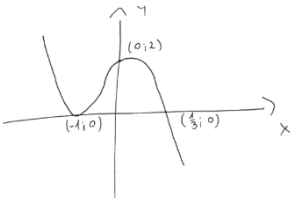
If there is any ambiguity regarding coordinates, the sketch takes precedence.

B1: Reflection in the *y*-axis. This requires a minimum on the negative *x*-axis and a maximum to the left of the *y*-axis in quadrant 2. If the maximum is on the *y*-axis score B0.
The curve must continue upwards to the left of the *y*-axis to reach a maximum in quadrant 2.
The curve must extend into quadrant 4 as shown.

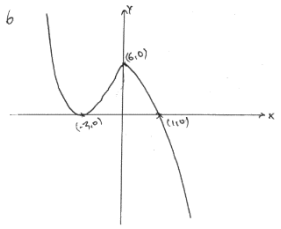
B1: Minimum at (−1, 0). Allow just −1 marked in the correct place and condone (0, −1) as long as it is marked in the correct place. Allow to score away from the sketch but must be identified correctly as (−1, 0) and correspond to the sketch.

B1: Passes through (0, 2) and passes through or touches $\left(\frac{1}{3}, 0\right)$. For both points, allow just the intercepts marked in the correct place and condone (2, 0) and/or $\left(0, \frac{1}{3}\right)$ as long as they are marked in the correct place. Allow to score away from the sketch but must be identified correctly as (0, 2) and $\left(\frac{1}{3}, 0\right)$ and correspond to the sketch.

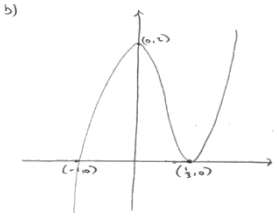
See examples below:



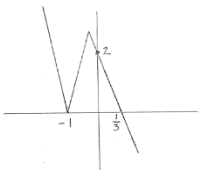
B0: Maximum in quadrant 2
B1: Minimum at (−1, 0)
B1: Passes through (0, 2) and $\left(\frac{1}{3}, 0\right)$



B0: Maximum on the *y*-axis (we would have tolerated the slight "pointy" maximum)
B0: Minimum at (−2, 0)
B0: Does not pass through (0, 2) and $\left(\frac{1}{3}, 0\right)$



B0: Not a reflection in the *y*-axis
B0: No minimum at (−1, 0)
B1: Passes through (0, 2) and touches $\left(\frac{1}{3}, 0\right)$



B0: Must be a curve – a reflection in the *y*-axis with the same shape.
B1: Minimum at (−1, 0)
B1: Passes through (0, 2) and $\left(\frac{1}{3}, 0\right)$