Question	Scheme	Marks
1	$\int\left(10x^4-\frac{3}{2x^3}-7\right)\mathrm{d}x=2x^5+\frac{3}{4}x^{-2}-7x+c$	M1A1A1
		(3)
		Total 3

Notes:

M1: For increasing one of the powers of x by 1 e.g. $x^4\rightarrow...x^5$ or $x^{-3}\rightarrow...x^{-2}$ or $7\rightarrow...7x$

A1: Any **two** correctly integrated terms, unsimplified or simplified.

Allow e.g. $\frac{10}{4+1}x^{4+1}$, $-\frac{3}{-4}x^{-3+1}$, $-7x^1$

A1: $2x^5+\frac{3}{4}x^{-2}-7x+c$ all correct and simplified in one expression and including "+ c".

Allow correct simplified equivalents for $\frac{3}{4}x^{-2}$ e.g. $0.75x^{-2}$, $\frac{3}{4x^2}$ but do **not** allow $-7x^1$ for $-7x$ or $\frac{2}{1}$ for 2

Award this mark once a correct simplified expression is seen and apply isw if necessary.

Condone poor notation e.g. spurious integral signs, "dx", $\frac{\mathrm{d}y}{\mathrm{d}x}=...$ and look for the correct expression.

Question	Scheme	Marks
2(i)(a)	$2^{n+3} = 2^n \times 2^3 = 8m$	B1
		(1)
(b)	$16^{3n} = \left(2^4\right)^{3n}$	M1
	$= 2^{12n} = \left(2^n\right)^{12} = m^{12}$	A1
		(2)

Notes:

(i)(a)

B1: For $8m$ (Condone 8M). Do **not** allow $2^3 m$.

(b)

M1: For writing 16^{3n} correctly as an expression involving a power of 2.

Examples: $\left(2^4\right)^{3n}$, $\left(2^2\right)^{6n}$, $\left(2^{3n}\right)^4$, $\left(2^n\right)^{12}$, $\left(2^{12}\right)^n$, 2^{12n}

A1: For m^{12} (Condone M^{12})

Correct answer only scores both marks.

(ii)	$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow x\sqrt{3}-x=3+\sqrt{3}\Rightarrow x\left(\sqrt{3}-1\right)=3+\sqrt{3}\Rightarrow x=...$	M1
	$=\dfrac{3+\sqrt{3}}{\sqrt{3}-1}\times\dfrac{\pm\left(\sqrt{3}+1\right)}{\pm\left(\sqrt{3}+1\right)}$	M1
	$=\dfrac{\pm\left(4\sqrt{3}+6\right)}{\pm\left(3-1\right)}=3+2\sqrt{3}$	A1
		(3)

Notes:

In this part, the A1 depends on both M marks

(ii)

M1: Collects *x* terms to one side, factorises and makes *x* the subject

It is for processing the given equation and reaching $x=\dfrac{\alpha+\beta\sqrt{3}}{\gamma+\delta\sqrt{3}},\ \alpha,\beta,\gamma,\delta\neq 0$

M1: Correct attempt to rationalise their denominator.

Not formally dependent but requires $\dfrac{...}{\gamma+\delta\sqrt{3}}=\dfrac{...}{\gamma+\delta\sqrt{3}}\times\dfrac{\gamma-\delta\sqrt{3}}{\gamma-\delta\sqrt{3}}$ or equivalent.

Sight of e.g. $\dfrac{3+\sqrt{3}}{\sqrt{3}-1}\times\dfrac{\sqrt{3}+1}{\sqrt{3}+1}$ is sufficient for this mark **but may be implied** by e.g.

$\dfrac{3+\sqrt{3}}{\sqrt{3}-1}=\dfrac{3\sqrt{3}+3+3+\sqrt{3}}{3-1}$

A1: For $3+2\sqrt{3}$ with at least one intermediate step e.g.

$\dfrac{3+\sqrt{3}}{\sqrt{3}-1}=\dfrac{3\sqrt{3}+3+3+\sqrt{3}}{3-1}$ or $\dfrac{3+\sqrt{3}}{\sqrt{3}-1}\times\dfrac{\sqrt{3}+1}{\sqrt{3}+1}=\dfrac{4\sqrt{3}+6}{2}$ or $\dfrac{3+\sqrt{3}}{\sqrt{3}-1}\times\dfrac{\sqrt{3}+1}{\sqrt{3}+1}=\dfrac{3\sqrt{3}+3+3+\sqrt{3}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}$

Allow $2\sqrt{3}+3$. It is for this expression and not for just *p* = 3, *q* = 2 unless $3+2\sqrt{3}$ is seen.

Example of insufficient work:

$$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow x\sqrt{3}-x=3+\sqrt{3}\Rightarrow x\left(\sqrt{3}-1\right)=3+\sqrt{3}\Rightarrow x=\frac{3+\sqrt{3}}{\sqrt{3}-1}$$

$$=\frac{3+\sqrt{3}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=3+2\sqrt{3}$$

Scores M1M1A0

Note that:

$$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow x\sqrt{3}-x=3+\sqrt{3}\Rightarrow x\left(\sqrt{3}-1\right)=3+\sqrt{3}\Rightarrow x=\frac{3+\sqrt{3}}{\sqrt{3}-1}=3+2\sqrt{3}$$

Scores M1M0A0

Alternatives for part (ii)

(ii) ALT 1	$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow\left(p+q\sqrt{3}\right)\sqrt{3}-3=p+q\sqrt{3}+\sqrt{3}\Rightarrow p\sqrt{3}+3q-3=p+\sqrt{3}+q\sqrt{3}$ $\Rightarrow p=q+1,\ 3q-3=p$	M1
	$\Rightarrow p=q+1,\ 3q-3=p\Rightarrow p=...,q=...$	M1
	$x=3+2\sqrt{3}$	A1

Notes:

M1: Substitutes $x=p+q\sqrt{3}$, collects terms, compares coefficients and forms 2 equations in p and q
It is for obtaining 2 equations of the form $\alpha p+\beta q=\gamma,\ \alpha,\beta,\gamma\neq 0$

M1: Solves simultaneously with evidence of algebra i.e. answers not just written down from a calculator.
Not formally dependent but requires the solution of 2 equations of the form $\alpha p+\beta q=\gamma,\ \alpha,\beta,\gamma\neq 0$

A1: For $3+2\sqrt{3}$. Allow $2\sqrt{3}+3$. It is for this expression and not for just p = 3, q = 2 unless $3+2\sqrt{3}$ is seen.

(ii) ALT 2	$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow 3x^2-6\sqrt{3}x+9=x^2+2\sqrt{3}x+3$ $\Rightarrow 2x^2-8\sqrt{3}x+6=0$	M1
	$x^2-4\sqrt{3}x+3=0\Rightarrow x=\dfrac{4\sqrt{3}\pm\sqrt{\left(4\sqrt{3}\right)^2-4\times1\times3}}{2}$	M1
	$\Rightarrow x=\dfrac{4\sqrt{3}\pm\sqrt{36}}{2}\Rightarrow x=2\sqrt{3}\pm3$ $x=2\sqrt{3}+3$	A1

Notes:

M1: Squares both sides. Must obtain a 3 term quadratic expression on both sides and collect terms to one side.

M1: Solves a 3TQ of the form $px^2+q\sqrt{3}x+r=0$ by a correct non-calculator method.

A1: For $3+2\sqrt{3}$. Allow $2\sqrt{3}+3$. It is for this expression and not for just p = 3, q = 2 unless $3+2\sqrt{3}$ is seen.
If any other answers are offered and clearly not rejected score A0.

(ii) ALT 3	$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow\left(x\sqrt{3}-3\right)\left(x-\sqrt{3}\right)=\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$ $\Rightarrow\sqrt{3}x^2-6x+3\sqrt{3}=x^2-3\Rightarrow\left(\sqrt{3}-1\right)x^2-6x+3\sqrt{3}+3=0$	M1
	$\left(\sqrt{3}-1\right)x^2-6x+3\sqrt{3}+3=0\Rightarrow x=\dfrac{6\pm\sqrt{36-4\left(\sqrt{3}-1\right)\left(3\sqrt{3}+3\right)}}{2\left(\sqrt{3}-1\right)}$ $=\dfrac{6\pm\sqrt{36-24}}{2\left(\sqrt{3}-1\right)}=\dfrac{6\pm\sqrt{12}}{2\left(\sqrt{3}-1\right)}=\dfrac{3\pm\sqrt{3}}{\sqrt{3}-1}=\dfrac{3\pm\sqrt{3}}{\sqrt{3}-1}\times\dfrac{\sqrt{3}+1}{\sqrt{3}+1}$	M1
	$=\dfrac{3\sqrt{3}+3\pm3\pm\sqrt{3}}{2}=2\sqrt{3}+3,\ \sqrt{3}$ $x=2\sqrt{3}+3$	A1

Notes:

M1: Multiples both sides by e.g. $\left(x-\sqrt{3}\right)$ and collects terms to one side.
Must obtain a 3 term quadratic expression on lhs and a 2 or 3 term quadratic expression on rhs.

M1: Solves a 3TQ of the form $\left(A+B\sqrt{3}\right)x^2+Cx+D+E\sqrt{3}=0,\ A,B,C,D,E\neq0$ by a correct non-calculator method **and** then attempts to rationalise the denominator as in the main scheme.

A1: For $3+2\sqrt{3}$. Allow $2\sqrt{3}+3$. It is for this expression and not for just p = 3, q = 2 unless $3+2\sqrt{3}$ is seen.
If any other answers are offered and clearly not rejected score A0.

		Total 6