Question Number	Scheme	Marks
3a	$\dfrac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}}\times\dfrac{2\sqrt{3}-\sqrt{5}}{2\sqrt{3}-\sqrt{5}}=\dfrac{16\sqrt{3}-8\sqrt{5}-2\sqrt{45}+\sqrt{75}}{12-5}$	M1
	e.g. $\dfrac{21\sqrt{3}-14\sqrt{5}}{7}$	dM1
	$3\sqrt{3}-2\sqrt{5}$	A1
		(3)
b	$\left(x+5\sqrt{3}\right)\sqrt{5}=40-2x\sqrt{3}\Rightarrow x\sqrt{5}+2x\sqrt{3}=40-5\sqrt{15}$	M1
	$\left(x=\right)\dfrac{40-5\sqrt{15}}{2\sqrt{3}+\sqrt{5}}$	A1
	$\left(x=\right)15\sqrt{3}-10\sqrt{5}$	A1ft
		(3)
		(6 marks)

(a) Note that $3\sqrt{3}-2\sqrt{5}$ with no working scores M0dM0A0

M1: Attempts to rationalise the denominator by multiplying both numerator and denominator by $k\left(2\sqrt{3}-\sqrt{5}\right)$, where k is an integer usually 1, and proceeds to a fraction such as $\dfrac{...\sqrt{3}\pm...\sqrt{5}\pm...\sqrt{45}\pm...\sqrt{75}}{...}$ or $\dfrac{...}{12-5}$ or $\dfrac{...}{7}$

but not for e.g. $\dfrac{...}{12+2\sqrt{15}-2\sqrt{15}-5}$ or $\dfrac{...}{\left(2\sqrt{3}\right)^2-\left(\sqrt{5}\right)^2}$ as these still have surds on the denominator.

Allow $\sqrt{45}$ and $\sqrt{75}$ to be written in terms of $\sqrt{3}$ and $\sqrt{5}$ as well.
Condone slips in multiplying out as well as miscopying errors.

Note $\dfrac{21\sqrt{3}-14\sqrt{5}}{7}$ with no intermediate working can still score M1 for the denominator.

Attempting to multiply by a multiple of $\dfrac{2\sqrt{3}+\sqrt{5}}{2\sqrt{3}+\sqrt{5}}$ is M0dM0A0

dM1: Attempts to simplify surds and may collect terms to achieve a fraction where

- the numerator is in terms of $\sqrt{3}$ and $\sqrt{5}$ only.
- the denominator is a multiple of 7 (which may be unsimplified)

It is dependent on the previous method mark.
If they have not fully multiplied out the numerator (or implied) then this mark cannot be scored. e.g.
$\dfrac{(8-\sqrt{15})(2\sqrt{3}-\sqrt{5})}{...}=\dfrac{16\sqrt{3}+8\sqrt{5}}{...}$ scores a maximum of M1dM0A0.

Note $\dfrac{21\sqrt{3}-14\sqrt{5}}{7}$ **with no previous working seen** scores M1dM0A0 because they have not shown any multiplying out of the brackets on the numerator.

Note $\dfrac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}}=\dfrac{16\sqrt{3}-8\sqrt{5}-2\sqrt{45}+\sqrt{75}}{7}=3\sqrt{3}-2\sqrt{5}$ is M1dM0A0 because they have not collected terms on the numerator (or changed them all to be in terms of $\sqrt{3}$ and $\sqrt{5}$ before simplifying to the final answer).

A1: $3\sqrt{3}-2\sqrt{5}$. The answer does not imply the method marks. Do not withhold this mark for slips in working such as invisible brackets provided they are recovered/implied by further work.

Note that a number of candidates are misreading $\frac{8-\sqrt{15}}{2\sqrt{3}+5}\times\frac{2\sqrt{3}-5}{2\sqrt{3}-5}=\frac{16\sqrt{3}-40-2\sqrt{45}+5\sqrt{15}}{12-25}$ this can score SC100 condoning one sign slip in their multiplying out of the brackets in the numerator.

(b) Note that $x=15\sqrt{3}-10\sqrt{5}$ with no working scores M0A0A0

M1: Multiplies out the brackets and isolates the *x* terms on one side. Condone for this mark if the surds are converted to rounded decimals.

e.g. $(\sqrt{5}+2\sqrt{3})x=5(8-\sqrt{15})$ scores M1

Alternatively divides both sides by $\sqrt{5}$ and isolates the *x* terms on one side.

e.g. $x+2x\frac{\sqrt{3}}{\sqrt{5}}=\frac{40}{\sqrt{5}}-5\sqrt{3}$

Condone slips in their working and invisible brackets, but there must be two terms on each side of the equation.

A1: $x=\frac{40-5\sqrt{15}}{2\sqrt{3}+\sqrt{5}}$ or exact equivalent (which cannot be $15\sqrt{3}-10\sqrt{5}$).

In the alternative method it may be seen as $x=\frac{\frac{40}{\sqrt{5}}-5\sqrt{3}}{1+2\frac{\sqrt{3}}{\sqrt{5}}}$ which can score this mark.

Do not accept proceeding from $(\sqrt{5}+2\sqrt{3})x=5(8-\sqrt{15})\Rightarrow x=15\sqrt{3}-10\sqrt{5}$ without any intermediate expression for *x*. This scores A0 as the use of calculator technology is not acceptable.
An expression in decimals only is A0.

A1ft: $x=15\sqrt{3}-10\sqrt{5}$ only (or exact simplified equivalent) e.g. $x=5\left(3\sqrt{3}-2\sqrt{5}\right)$ This mark cannot be awarded without the previous A mark being scored.

Follow through on their part (a) answer of the form $a\sqrt{3}+b\sqrt{5}$ so award for $5a\sqrt{3}+5b\sqrt{5}$ oe including $5\left(a\sqrt{3}+b\sqrt{5}\right)$ where *a* and *b* may be fractions. Isw once they have achieved the correct answer.

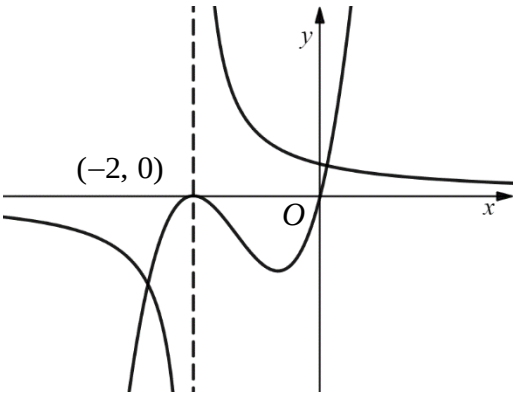
Note: If a candidate has an unsimplified answer in part (a) e.g. $\frac{21\sqrt{3}-14\sqrt{5}}{7}$ then do not withhold this mark in part (b) if they proceed to e.g. $\frac{105\sqrt{3}-70\sqrt{5}}{7}$ as they have already been penalised once for not giving an answer in simplest form.

Alternative method – squaring both sides (send to review if unsure)

M1: Attempts to square both sides and proceeds to a three term quadratic in *x* with terms collected on one side of the equation (condone slips in multiplying out and collecting terms.

A1: As above in main scheme

A1ft: As above in main scheme

Question Number	Scheme	Marks
4a	$x=-2$	B1
		(1)
b	$x^3+4x^2+4x=x(x+2)^2$	M1A1
		(2)
c		B1B1B1
		(3)
d	2 as the graphs intersect (each other) twice (since $(x+2)(x^3+4x^2+4x)=1$ is the same as $x^3+4x^2+4x=\frac{1}{x+2}$)	B1
		(1)
		(7 marks)

Note: Check for answers by the questions or on the Figure (rather than the diagram). If there is a contradiction between the answers by the question and the main body of the solution then the main body of the solution takes precedence.

(a)

B1: $x=-2$ only. Accept labelled on the diagram but do not accept just -2. If other equations are given (other than $y=0$) then B0.

Beware that $x=-2$ is one of the solutions to the cubic so the equation of the asymptote must be seen in (a), on the diagram or by the question.

(b)

M1: Attempts to take out a linear factor eg $x(x^2\pm4x\pm4)$ or $(x+2)(x^2\pm2x)$. May be implied by the correct answer.

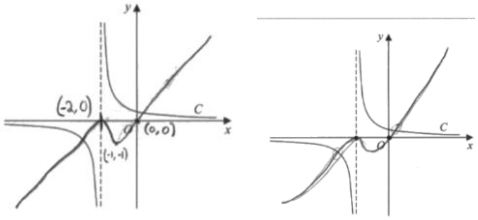
A1: $x(x+2)^2$ or $x(x+2)(x+2)$. Accept $(x+2)(x+2)x$ and condone a missing trailing bracket on the final linear factor e.g. $x(x+2)(x+2$ but $x(x+2^2$ is A0.

Condone $(x+0)(x+2)^2$. Isw once a correct expression is seen and ignore a spurious = 0

(c) **Note: If there are multiple attempts then score the highest one**

B1: Sketches a positive cubic anywhere on a set of axes. Do not penalise poor curvature provided the intention is clear and ignore if the curvature looks asymptotic. The cubic does not need to have two turning points for this mark.

Examples of acceptable cubic shapes (including where poor curvature would be condoned):



(If you are unsure then send to review)