Question	Scheme	Marks
6(a)	$\left(r-\frac{1}{r}\right)^2 = r^2 - r\times\frac{1}{r} - r\times\frac{1}{r} + \frac{1}{r^2}$	**M1**
	$= r^2 + \frac{1}{r^2} - 2$	**A1**
		(2)
(b)	$\frac{1}{3+2\sqrt{2}} = \frac{1}{3+2\sqrt{2}}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}$	**M1**
	$= \frac{3-2\sqrt{2}}{3^2-\left(2\sqrt{2}\right)^2} = 3-2\sqrt{2}$	**A1**
		(2)
(b) ALT	$\frac{1}{3+2\sqrt{2}} = p+q\sqrt{2} \Rightarrow 1 = \left(p+q\sqrt{2}\right)\left(3+2\sqrt{2}\right) = 3p+4q+2p\sqrt{2}+3q\sqrt{2}$ $\Rightarrow \begin{matrix} 3p+4q=1 \\ 2p+3q=0 \end{matrix}$	**M1**
	$\Rightarrow \begin{matrix} 3p+4q=1 \\ 2p+3q=0 \end{matrix} \Rightarrow \begin{matrix} 6p+8q=2 \\ 6p+9q=0 \end{matrix} \Rightarrow q=-2 \Rightarrow 3p-8=1 \Rightarrow p=3$ $\frac{1}{3+2\sqrt{2}} = 3-2\sqrt{2}$	**A1**
		(2)
(c)	$\left(\sqrt{3+2\sqrt{2}} - \frac{1}{\sqrt{3+2\sqrt{2}}}\right)^2 = 3+2\sqrt{2}+\frac{1}{3+2\sqrt{2}}-2$	**M1**
	$= 3+2\sqrt{2}+3-2\sqrt{2}-2 = \ldots(=4)$	**dM1**
	so $\sqrt{3+2\sqrt{2}} - \frac{1}{\sqrt{3+2\sqrt{2}}} = 2$	**A1**
		(3)
(c) Alt	$\sqrt{3+2\sqrt{2}} - \frac{1}{\sqrt{3+2\sqrt{2}}} = 2 \Rightarrow 3+2\sqrt{2}-1 = 2\sqrt{3+2\sqrt{2}}$	**M1**
	$\Rightarrow \left(2+2\sqrt{2}\right)^2 = 4\left(3+2\sqrt{2}\right)$	**dM1**
	$\Rightarrow 4+8\sqrt{2}+8 = 12+8\sqrt{2}$ ✓ Hence true	**A1**
		(3)
		(7 marks)

Notes:

(a)

M1: Expands the bracket to obtain 3 or 4 terms with at least 2 correct which may be unsimplified. Allow use of a different variable e.g. x for this mark.

A1: Correct simplified expansion in terms of r. Accept terms in any order.

Accept e.g. $r^2 + r^{-2} - 2$ and accept $r^2 + \left(\frac{1}{r}\right)^2 - 2$. Do **not** isw and mark their final answer.

Note that if a correct simplified expression is seen and they then re-write the expression correctly in a different way then this mark should be awarded provided their expression is correct.

(b)

M1: Correct process to rationalise the denominator, so look for multiplying numerator and denominator by $3-2\sqrt{2}$ or any multiple of this expression. No processing is required so just look for the statement as shown above.

A1: Shows an intermediate line before obtaining $3-2\sqrt{2}$

Examples of an acceptable intermediate line are:

$$\frac{3-2\sqrt{2}}{3^2-\left(2\sqrt{2}\right)^2},\ \frac{3-2\sqrt{2}}{9-8},\ \frac{3-2\sqrt{2}}{9-6\sqrt{2}+6\sqrt{2}-\left(2\sqrt{2}\right)^2},\ \frac{3-2\sqrt{2}}{9-6\sqrt{2}+6\sqrt{2}-8},\ \frac{3-2\sqrt{2}}{1}$$

Do **not** allow $\frac{3-2\sqrt{2}}{1}$ as the final answer.

(b) ALT

M1: Sets $\frac{1}{3+2\sqrt{2}} = p+q\sqrt{2},$ multiplies up, compares rational and irrational parts to produce 2 equations in p and q.

A1: Solves simultaneously **showing working** and obtains $3-2\sqrt{2}$.

(c)

M1: Applies their result from (a) or the correct result to $\left(\sqrt{3+2\sqrt{2}} - \frac{1}{\sqrt{3+2\sqrt{2}}}\right)^2$

dM1: Depends on the first mark. Applies their result from (b) and cancels $\sqrt{2}$ terms to achieve a constant.

A1: Reaches the correct answer from correct work. **There is no requirement to justify the positive square root.**

(c) ALT

M1: Multiplies equation through by $\sqrt{3+2\sqrt{2}}$ and applies $\sqrt{3+2\sqrt{2}} \times \sqrt{3+2\sqrt{2}} = 3+2\sqrt{2}$

dM1: Depends on the first mark. Squares both sides. May or may not have cancelled the 2 before squaring.

A1: Achieves similar expression for both sides and gives a (minimal) conclusion.

Note that there will be other methods for 6(c) e.g.:

$$\sqrt{3+2\sqrt{2}}-\frac{1}{\sqrt{3+2\sqrt{2}}}=\frac{3+2\sqrt{2}-1}{\sqrt{3+2\sqrt{2}}}$$

M1: Attempts common denominator and applies $\sqrt{3+2\sqrt{2}}\times\sqrt{3+2\sqrt{2}}=3+2\sqrt{2}$

$$\left(\frac{2+2\sqrt{2}}{\sqrt{3+2\sqrt{2}}}\right)^2=\frac{12+8\sqrt{2}}{3+2\sqrt{2}}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}=\frac{36-32}{9-8}=4$$

dM1: Squares and attempts to rationalise the denominator

So $\sqrt{3+2\sqrt{2}}-\frac{1}{\sqrt{3+2\sqrt{2}}}=2*$

A1*: Fully correct work

Question	Scheme	Marks
7(a)	$0=10-2x \Rightarrow x=5$ or $y=2, y=10-2x \Rightarrow x=4$	B1
	Examples: $\frac{1}{2}\times 2\left(5+4-a\right)=\frac{27}{4}$ or $\frac{1}{2}\times 2\left(5+4-\frac{2}{k}\right)=\frac{27}{4}$ Trapezium or $\frac{1}{2}\times 2a+\frac{1}{2}\times 2\left(5-a+4-a\right)=\frac{27}{4}$ or $\frac{1}{2}\times 2\times\frac{2}{k}+\frac{1}{2}\times 2\left(5-\frac{2}{k}+4-\frac{2}{k}\right)=\frac{27}{4}$ Triangle + Trapezium or $\frac{1}{2}\times 2a+2\left(4-a\right)+\frac{1}{2}\times 1\times 2=\frac{27}{4}$ or $\frac{1}{2}\times 2\times\frac{2}{k}+2\left(4-\frac{2}{k}\right)+\frac{1}{2}\times 1\times 2=\frac{27}{4}$ Triangle + Rectangle + Triangle or $\frac{1}{2}\times 5\times 2+\frac{1}{2}\left(4-a\right)\times 2=\frac{27}{4}$ or $\frac{1}{2}\times 5\times 2+\frac{1}{2}\left(4-\frac{2}{k}\right)\times 2=\frac{27}{4}$ 2 Triangles	M1
	$\Rightarrow k=\frac{8}{9}, a=\frac{9}{4}$	A1 A1ft
		(4)
(b)	Two of $y\geqslant "\frac{8}{9}" x,\ y\leqslant 10-2x, x>"\frac{9}{4}"$	M1
	All three of $y\geqslant \frac{8}{9}x,\ y\leqslant 10-2x, x>\frac{9}{4}$	A1
		(2)
		(6 marks)

Notes:

(a)

B1: Correct x intercept for $y=10-2x$ or correct x coordinate for intersection of $y = 2$ with $y=10-2x$. May be seen on the diagram or in a sketch or implied by their working.

M1: Correct method for area and sets equal to $\frac{27}{4}$ to form an equation in a or k. The $2\times\frac{1}{2}$ may be implied. May use e.g. trapezium, triangle + trapezium, triangle + rectangle + triangle or 2 triangles. See above for some examples. Note that with a correct "5" and "4" the area is $9-a$ or $9-\frac{2}{k}$.

A1: Correct value for a or k as a single value not a calculation.

A1ft: Correct values for a and k. Correct values or follow through using $ak=2$ from their first value to the second **provided the M1 is scored**. Allow exact equivalents or exact ft equivalents so if correct allow e.g. $k=0.\dot{8}$, $a=2.25$ or $2\frac{1}{4}$