| Question Number | Scheme | Marks |
|---|---|---|
| **3(i)** | $\sqrt{8}=2\sqrt{2}$ seen anywhere in the solution (see notes) | B1 |
| | $\left(x+\sqrt{2}\right)^2+\left(3x-5\sqrt{8}\right)^2$<br>$=x^2+2x\sqrt{2}+2+9x^2-30x\sqrt{8}+25\times 8$ | M1 |
| | $=10x^2-58x\sqrt{2}+202$ | A1 |
| | | **(3)** |
| **(ii)** | $\sqrt{3}\left(4y-3\sqrt{3}\right)=5y+\sqrt{3}$<br>$\Rightarrow 4\sqrt{3}y-9=5y+\sqrt{3}$<br>$\Rightarrow 4\sqrt{3}y-5y=9+\sqrt{3}$ | M1 |
| | $\Rightarrow y=...$ or $\Rightarrow ky=...$<br>eg $y="\frac{9+\sqrt{3}}{4\sqrt{3}-5}"$ or $"23"y="9+\sqrt{3}"$ | dM1 |
| | $y=\frac{9+\sqrt{3}}{4\sqrt{3}-5}\times\frac{4\sqrt{3}+5}{4\sqrt{3}+5}\Rightarrow y=\frac{...}{"23"}$ | ddM1 |
| | $y=\frac{57}{23}+\frac{41}{23}\sqrt{3}$ (or $y=2\frac{11}{23}+1\frac{18}{23}\sqrt{3}$) | A1 |
| | | **(4)** |
| | **(ii) Alternative 1:** | |
| | $\sqrt{3}\left(4p+4q\sqrt{3}-3\sqrt{3}\right)=5\left(p+q\sqrt{3}\right)+\sqrt{3}$<br>$\Rightarrow 4p\sqrt{3}+12q-9=5p+5q\sqrt{3}+\sqrt{3}$<br>$\Rightarrow 4p=5q+1,\ 12q-9=5p$ | M1<br>dM1 |
| | $\Rightarrow 4p=5q+1,\ 12q-9=5p$<br>$\Rightarrow p=...,\ q=...$ | ddM1 |
| | $y=\frac{57}{23}+\frac{41}{23}\sqrt{3}$ (or $y=2\frac{11}{23}+1\frac{18}{23}\sqrt{3}$) | A1 |
| | | **Total 7** |

**Note solutions relying on calculator technology are not acceptable.**
**(i)**

B1: May be stated in the margin or seen/implied anywhere in the solution.
**It does not need to be explicitly stated.**

Eg. sight of $30\sqrt{8}\rightarrow 60\sqrt{2}$ is fine as is $\left(3x-5\sqrt{8}\right)^2\rightarrow ...x^2-60\sqrt{2}+...$

M1: Expands both brackets so look for:
$x^2+ax\sqrt{2}+2+bx^2-cx\sqrt{8}+d$ or $x^2+ax\sqrt{2}+2+bx^2-cx\sqrt{2}+d$
where *a*, *b*, *c* and *d* are non-zero. **The *x* terms do not need to be collected to score this mark.**

A1: $10x^2-58x\sqrt{2}+202$ or equivalent isw after a correct answer. $58\sqrt{2}x$ is acceptable

Eg $x^2+2x\sqrt{2}+2+9x^2-30x\sqrt{8}+25\times 8 \Rightarrow 10x^2-58x\sqrt{2}+202$ scores B1M1A1
as $\sqrt{8}=2\sqrt{2}$ is correctly seen by the correct collection of the terms.
Eg $x^2+2x\sqrt{2}+2+9x^2-30x\sqrt{8}+25\times 8 \Rightarrow 10x^2-60x\sqrt{2}+202$ scores B0M1A0
as $\sqrt{8}=2\sqrt{2}$ is not correctly seen or implied as the coefficient of $x$ is incorrect

**(ii) On EPEN this is M1A1M1A1 we are marking this M1M1M1A1**

M1: Attempts to multiply out and isolates the two $y$ terms on one side of the equation.
**Condone sign slips only in their rearrangement.**

dM1: Attempts to make $y$ or $ky$ the subject (where $k$ is an integer or fraction which would simplify to an integer). Score for $y=\mathrm{f}(x)$ or $ky=\mathrm{g}(x)$ where the function includes $\sqrt{3}$ but it cannot be scored for directly proceeding to $y=\frac{57}{23}+\frac{41}{23}\sqrt{3}$ or $y=\frac{57+41\sqrt{3}}{23}$.
It is dependent on the previous method mark.
Note $y\left(4\sqrt{3}-5\right)=9+\sqrt{3} \Rightarrow y=\frac{57}{23}+\frac{41}{23}\sqrt{3}$ is M1dM0ddM0A0

ddM1: Proceeds to $y=....$ with a rational denominator. The denominator does not need to be simplified.
It is dependent on the previous method mark. They must have shown the step of either rationalising the denominator or showing how they achieve $ky=...$ and proceeds to $y=...$

A1: Correct answer in the correct form with all stages of working seen
- Collects $y$ terms on one side of the equation
- Makes $y$ the subject
- Rationalises and proceeds to the correct answer. Somewhere in their solution they will have had to have multiplied two brackets involving surds together and they must have shown this being multiplied out (calculators are not allowed)
  eg $\frac{9+\sqrt{3}}{4\sqrt{3}-5}\times\frac{4\sqrt{3}+5}{4\sqrt{3}+5}=\frac{36\sqrt{3}+45+12+5\sqrt{3}}{23}$

Condone solutions with invisible brackets to score full marks, provided the general method is sound and the answer has not just come from a calculator or incorrect working.
Do not accept $y=\frac{57+41\sqrt{3}}{23}$

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**Alt(ii)1 (main scheme alternative)**

M1: Substitutes $y=p+q\sqrt{3}$ expands, collects terms. Condone sign slips.

dM1: Compares rational/irrational parts to form two equations. It is dependent on the previous method mark.

ddM1: Solves 2 linear equations in $p$ and $q$ using an acceptable method. They cannot just state the values. Condone slips in their working. It is dependent on the previous method mark.

A1: $y=\frac{57}{23}+\frac{41}{23}\sqrt{3}$ (or $y=2\frac{11}{23}+1\frac{18}{23}\sqrt{3}$) with full working shown.

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**Alt(ii)2** Squaring and completing the square

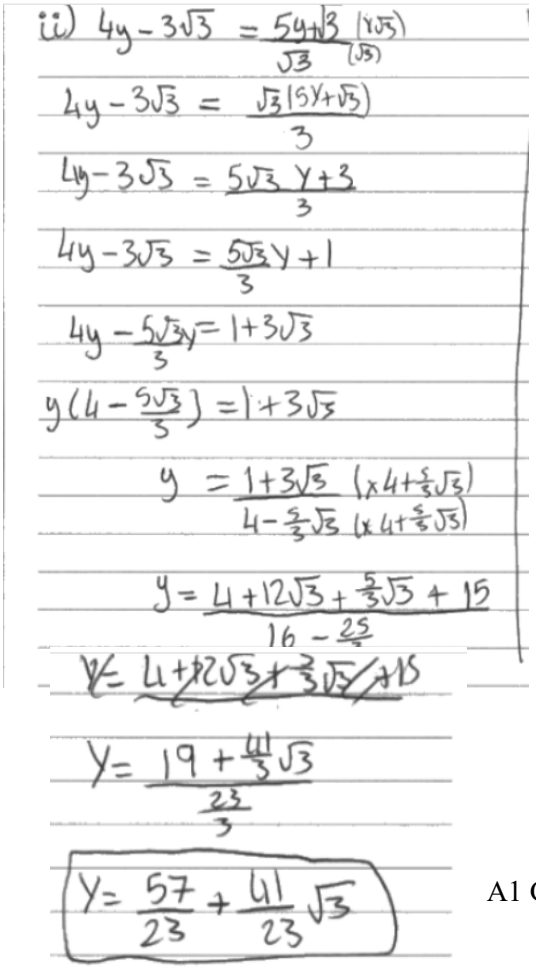
M1: Squares both sides and multiplies out brackets (condone slips)
eg. $(48y^2-72\sqrt{3}y+81=25y^2+3+10\sqrt{3}y)$

dM1: Rearranges to form a 3TQ = 0 $(23y^2-82\sqrt{3}y+78=0)$. It is dependent on the previous method mark.

ddM1: Attempts to complete the square and proceeds to make $y$ the subject. It is dependent on the previous method mark.

A1: $y = \frac{57}{23} + \frac{41}{23}\sqrt{3}$ (or $y = 2\frac{11}{23} + 1\frac{18}{23}\sqrt{3}$)

**Alt(ii)3 Dividing by $\sqrt{3}$, then rationalising the denominator and collecting terms:**



M1 for isolating $y$ terms on one side of the equation

dM1 for making the $y$ the subject

ddM1 for proceeds to $y$=… with a rational denominator

A1 Correct answer with full working shown

| Question Number | Scheme | Marks |
|---|---|---|
| **4(a)** | $x+y=6,\ y=6x-2x^2+1$<br>$\Rightarrow 6-x=6x-2x^2+1$<br>$\Rightarrow 2x^2-7x+5=0$ oe | M1 |
| | $2x^2-7x+5=0\Rightarrow(2x-5)(x-1)=0$<br>$\Rightarrow x=\frac{5}{2},\ 1$ | M1 |
| | $x="\frac{5}{2}"\Rightarrow y=\frac{7}{2}$<br>or $x="1"\Rightarrow y=5$ | dM1 |
| | $(1,\ 5)$ and $(2.5,\ 3.5)$ | A1 |
| | | **(4)** |
| **(b)** | $y\geqslant 6x-2x^2+1$ oe | M1 |
| | $x+y\leqslant 6$ oe | |
| | $x\geqslant a$ where $1\leqslant a\leqslant 2.5$ | A1 |
| | $(\text{or } a\leqslant x\leqslant b \text{ where } 1\leqslant a\leqslant 2.5,\ b\geqslant 6)$ | A1 |
| | $y\geqslant 0$ ( or $0\leqslant y\leqslant c$ where $c\geqslant 3.5$ ) | |
| | **Allow strict or non-strict inequalities** | |
| | | **(3)** |
| | | **Total 7** |

**Note solutions relying on calculator technology are not acceptable so a complete method must be shown**

(a) **Answers only 0/4**

M1: Uses the line and the curve to obtain a 3TQ in $x$ or $y$ equal to zero (may be implied by solving their 3TQ). Condone slips in their rearrangement.

M1: Solves their 3TQ which must be different to $6x-2x^2+1=0$ by an acceptable method (factorising, formula or completing the square). They cannot state the roots without seeing a correct line of intermediate working. Allow recovery and condone lack of $3TQ=0$
If they use the quadratic formula, the values must be embedded in the formula before proceeding to the roots.
If they factorise then do not accept $2x^2-7x+5=0\Rightarrow(x-\frac{5}{2})(x-1)=0\Rightarrow x=...$
Condone $-2x^2+7x-5=0\Rightarrow(2x-5)(x-1)=0\Rightarrow x=...$

dM1: Solves for $y$ for at least one of their values of $x$ or alternatively solves for $x$ for at least one of their values of $y$. It is dependent on the previous method mark.

A1: $(1,\ 5)$ and $(2.5,\ 3.5)$ which do not need to be paired. May be seen as $x=...,\ y=...$. But cannot be awarded if they are incorrectly paired in their final answer. Condone omission of brackets around the coordinates. Ignore if they have assigned coordinates as $P$ and $Q$. Cannot be awarded if $x$ and $y$ are the wrong way round or if given as $(1,\ 3.5)$ and $(2.5,\ 5)$